

Self Affine Nature of Freeform Parametric Bezier Curves For Tool Path Generation

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Abstract: Beziers contribution to computer graphics has covered the road for CAD software. His developments serve as an entry gate into learning about modern computer graphics with new mathematical object known as a spline, or a smooth curve specified in terms of a few points. The importance of spline in design field may be visualized through the design concept of aircraft wings and car body design.

There has been a great deal of research on the problem of freeform synthesis. Despite rapid advances in the synthesis of tool motions in the trajectory, there has been much less research on the issue of fine-tuning for the definition the trajectory to achieve desired smoothness on the work piece. The quality of trajectory will depend upon the clarity of curve. Based upon the lacuna of publication of Bezier curve through MATLAB-10, and the discussion made with industrialized persons, it has been decided to represent the geometry of the trajectory through a suitable MATLAB program that will help full to researcher for designing free form curve. So, this paper presents the modeling of Bezier curve through the parametric concept to define the geometry of the trajectory. The curve has been tested in MATLAB-10 through the proposed program. The control points are the true points which are filtered from the imported IGES neutral file format exported through CATIAV5. Later the shape of the curve is changed globally by changing their control point coordinate value and represented graphically in a clear manner.

Keywords : IGES, Parametric curve, Bernstein function, Bezier technique, spider net

1. INTRODUCTION

In automotive (and other) design scenarios, many shapes are based on feature curves. Ultimately, feature curves will be used to define surfaces, such as car hoods or roofs, or other industrial product design surfaces. Freeform modeling is inevitable and core issue in design, analysis and production. The ability to create curves and surfaces is a vital part of applications such as illustration, design and analysis. But working with curves and surfaces precisely is challenging, because they are hard to represent and describe, and hence difficult to specify and manipulate interactively. One of the difficulties stems from the fact that manipulation of geometry in the computer is still rather far from being rightfully called CAD, since it is the user who often has to manipulate a large number of variables (e.g. control points) in order to produce desired geometric properties. The manipulation work starts with the parametric data so extracted from the neutral file used in data exchange process.

be a major concern for many firms. Companies typically select from direct translators (where files are read and written in their native data sets), international standard file formats such as STEP, IGES, etc., or from various software that runs from a common geometry to produce machine-independent geometry. Geometric models are computational structures that capture the spatial aspects of the objects of interest for an application.

Curve design is a challenging job for designer to design curves and surfaces in computer graphics, computer aided design and computer aided geometric modeling fields. Beziers contribution to computer graphics has covered the road for CAD software. There has been a great deal of research on the problem of freeform motion synthesis. Despite rapid advances in the synthesis of tool motions in the trajectory, there has been much less research on the issue of fine-tuning for the definition of tool motion on the trajectory to achieve desired smoothness on the work piece. The quality of trajectory will depend upon the clarity of curve. Ever more frequently the modern industry is manufacturing parts presenting curves and complex surfaces. On one hand this need has promptly been met by both the CAD and the CAM systems, through parametric curves. This paper presents the modeling of Bezier curve through the parametric concept to define the geometry of the trajectory. The curve has been tested in MATLAB-10 through the proposed program. Individual program has been written for higher degree and higher order of the curve. The control points are the true points which are filtered from the IGES neutral file format exported through CATIAV5. Later the shape of the curve is changed globally by changing their control point coordinate

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value and represented graphically in a clear manner. Based upon the lacuna of publication of Bezier curve through MATLAB-10, and the discussion made with industrialized persons, it has been decided to represent the geometry of the trajectory through a suitable MATLAB program which will help full to the researcher for designing free form curves.

2. LITERATURE RIEW:

Curve and surface design is an important tool in computer graphics, animation, and computer aided design. It requires mathematics and geometric concept even though many curve design are intuitive. As a result, it has been a challenging job in curve and surface design for shape modeling and freeform shape manufacturing. Shape creating operations depend on the "tool" that moves on the "workpiece" to produce desired shape as per the design parameter of the product. The tool operates on the workpiece as dictated by the trajectory of the tool, resulting in the manufactured shape [1, 2]. Often, the geometric and functional properties of the end product are dependent on the trajectory of the tool. In general, the trajectory of the tool is defined as the Cartesian space path of the tool relative to the workpiece. The manufacturing part is represented by mathematical curves and surfaces.

Currently, CAD tools have been widely used in the production and manufacture field. The body shapes of automobiles are presented by freeform surface. The most common free form shape description is parametric surfaces. Polynomial parametric surfaces are called sculptured surfaces. The terms "sculptured surface", "curved surfaces", "free-form surfaces" and sometimes simply "surfaces" are used interchangeably. Synthetic curves are the ones that are described by a set of data points (control points) such as splines and Bezier curves. Synthetic curves use parametric equations to synthesize the curve from input geometric information, which are more suitable for curve design.

Bezier basis with shape parameter is constructed by an integral approach. Bezier basis curves with shape parameter have most properties of Bernstein basis and the Bezier curves. Moreover the shape parameter can adjust the curve's shape with the same control polygon. Many shapes are based on feature curves which have to have "perfect" shape in order to produce aesthetically pleasing surfaces [4, 5, 6]. The Bezier curve representation is one that is utilized most frequently in computer graphics and geometric modeling.

A designed part [7, 8] can be represented using Bezier curves and surfaces. In parametric surface if one of the parameters reaches zero or one, the curve becomes exactly one of the

boundary curves surrounding the surface. If both parameters are held constant, a point is specified on the surface patch. Parametric based tool path is frequently used as a finishing tool path for machining [9, 10]. Because the tool path is driven directly along the u-v parametric curves of the surface itself. MATLAB [4,5, 11, 12] is a high-performance language for technically computing the curve required for CAGD.

Critical review of literatures pin pointed towards the lacuna of simplicity of formation of freeform curve. For its easiest description the present research describes the self-affine nature of parametric Bezier curves in detail along with its self-affine properties that can be extended to other types of polynomial. In this paper the knowledge has been restricted to cubic Bezier curves only. Also this paper introduces a new user-friendly MATLAB based-on program for dealing with the most important curves appearing in CAGD, as Bézier and B-splines curves.

3. FEATURE EXPOSITION VS PARAMETRIC EXPOSITION OF FREEFORM SHAPE

Feature modeling has become the fundamental design paradigm for Computer-Aided Design (CAD) systems. Most current feature modeling systems is freeform curve and surface modeling. Freeform feature modeling is a relatively new area of interest in CAD research. A feature is treated as a specific shape in the design interface (at instance level), it is defined (at feature class level) as a generic shape associated with the semantics of its engineering meaning. This includes functional attributes, expressing properties of the feature. **Parameters** of a freeform surface feature can generally be categorized on the basis of their influence on the shape: positioning and shape parameters, as shown in Fig-1. A positioning parameter can be either a reference or an auxiliary parameter. The former is a global datum used to position the whole feature, whereas the latter is a local datum used to position only one or more entities making up the feature shape. A freeform surface feature has one reference parameter, e.g. the key reference point in Fig-1(a.1.). Any one point can be chosen as key reference point as per requirement. Auxiliary parameters are always specified relative to a reference parameter.

4. DESCRIPTION OF THE NEUTRAL FILE "IGES":-

An IGES file is a sequential file consisting of a sequence of records. The file formats treat the product definition to be exchanged as a file of entities, each entity being represented in a standard format, to and from which the native

representation of a specific CAD/CAM system can be mapped. IGES file

An IGES file consists of five sections which must appear in the following order: Start section, Global section, Directory Entry (DE) section, Parameter Data (PD) section, and Terminate section, as shown in Figure 2. The role of these sections is summarized in the following subsections.

4.1. Basic IGES entities and their analysis:

IGES is based on the concept of entities. Entities could range from simple geometric objects, such as points, lines, plane, and arcs, to more sophisticated entities, such as subfigures and dimensions. Entities in IGES are divided in three categories: (a) Geometric entities: such as arcs, lines, and points that define the object (b) Annotation entities: such as dimensions and notes that aid in the documentation and visualization of the object (c) Structure entities: Those define the associations between other entities in IGES file.

First select the "sketch" icon of CATIA-V5 and then choose any one plane out of three plane: XY-plane, YZ- plane, ZX-plane. As a result grid will open for that plane. Now required sketch can be drawn by choosing the suitable icon: - line, circle, spline, planer patch etc. from the tool bar. After completion of the sketch the user has to go through constraint fixation text, so that the sketch will fix in its position. The constraint may be chosen by fixation of any set of entity as per the available constraint entities like:- distance, length, angle, radius/diameter, symmetry, fix, tangency, parallelism, perpendicular, horizontal, vertical etc for its fastening. After this, the sketch will be saved through its neutral format ".igs" mode as shown in the figure-2 for the exporting file. When the exported file will be imported by the user then the total sketch will be visualized as its digital code. Some of the IGES entities have been illustrated below as per their requirement for this research work.

5 . PARAMETRIC CURVE:

Curves and surfaces are specified by the user in terms of points and are constructed in an interactive process. The user starts by entering the co-ordinates of points, either by extracting from the IGES file of a product sketch or the data points of an available product so collected by the CMM

machine. After the curve has been drawn, the user may want to modify its shape by moving, adding, or deleting points.

A mathematical function $y=f(x)$ is the **explicit** representation and can be plotted as a curve. The **implicit** representation of a curve has the form $f(x, y) = 0$. It can represent multivalued curves (more than one y value for an x value). The explicit and implicit curve representation can be used only when the function is known. In practical applications-where complex curves such as the shape of a car are needed then a different approach is required. Such approach for the curve representation used in practice is called **parametric representation**. A two dimensional parametric curve has the form $P(t) = (f(t), g(t))$ or $P(t) = (x(t), y(t))$. The function f and g become the (x, y) coordinates of any point on the curve, and the points are obtained when the parameter " t " is varied over a certain interval $[a,b]$, normally $[0,1]$. A simple example of a 2D parametric curve is $P(t) = (2t-1, t^2)$. When " t " is varied from **0 to 1**, the curve proceeds from the initial point $P(0)=(-1,0)$ to the final point $P(1)=(1,1)$. The x -coordinate is linear in " t " and the y -coordinate varies as $(t)^2$.

The first derivative $\frac{dP(t)}{dt}$ is denoted by $P'(t)$ or by \dot{P} . This derivative is the tangent vector to the curve at any point. The derivative is a vector and not a point because it is the limit of the difference $\{(P(t+\Delta) - P(t)) / \Delta\}$, and the difference of points is a vector. As a vector, the tangent possesses a direction (the direction of the curve at the point) and a magnitude (Which indicates the speed of the curve at the point). The tangent however is not the slope of the curve, because the tangent is a pair of numbers, whereas the slope is a single number.

One example of parametric curve $y=\sin(t)$ has been taken for testing through MATLAB program. Where, $0 \leq t \leq 2\pi$. The form of solid cylindrical pipe shape in the form of $\sin(t)$ is presented on the fig:- 3.1. as per the output data.

5.1. The Bernstein Form Of The Bezier Curve

The first approach to the Bezier curve expresses it as a weighted sum of the points. Each control point is multiplied by a weight and the products are added. $P_0, P_1, P_2, \dots, P_n$

are control points and the weights by B_i . The result, $P(t)$, depends on the parameter t .

The blending functions are termed as Bernstein polynomials and are defined by

$$B_{n,i}(t) = \binom{n}{i} t^i (1-t)^{(n-i)},$$

$$\text{Where, } \binom{n}{i} = \frac{n!}{i!(n-i)!}$$

The Bezier curve expression in the form of weighted sum is

$$P(t) = \sum_{i=0}^n P_i B_i;$$

Where, $0 \leq t \leq 1$

MATLAB program have been prepared in such a manner that with change in parametric value it gives the x-, y- and z-coordinate of the locus of points through which trajectory will pass.

Fig:-3.2. shows the behavior of Bernstein function and fig.:5.1. presents the locus of points along with Bezier trajectory for tool movement.

Table:-1 Generated locus of Cartesian point for defining the trajectory by taking true control points P0, P1, P2 and P3 from

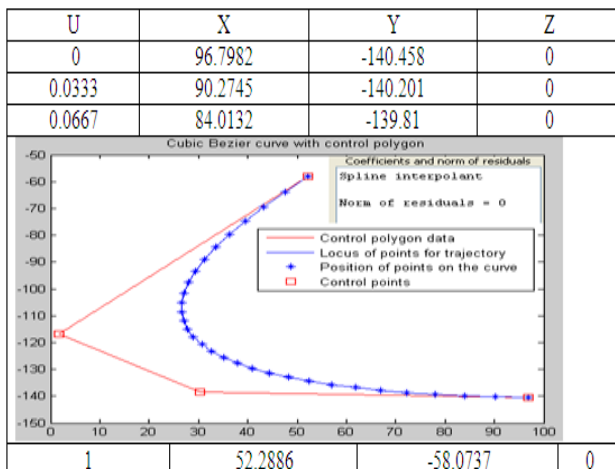


Fig:-5.1. Locus of points along with Bezier trajectory for tool movement.

For the control points $P_0 = (0; 0)$; $P_1 = (0.5; 1)$; $P_2 = (2; 1.2)$; $P_3 = (3; 0)$, the corresponding modified Bezier curves are shown on Fig: 5.3.

Table: 2

Given Tolerance	Deviation		Distance (s)	Locus of points to define the trajectory
0.005	0.0007 (Max.)	0.0 (Min.)	0.0816	26

The cubic Bezier curve can be represented by four control points. Fig.5.4 shows a cubic Bezier curve with four control points generated by the de Casteljau algorithm using MATLAB and cutter contact points so calculated are 26 points only. Maximum and minimum deviation between the designed curve and the linear interpolation of the CC points are 0.000694 in and 0 in with a given tolerance of 0.005 ins. The result is summarized in Table 2.

6.RESULT ANALYSIS:

In general, the parametric curves used in computer graphics and computer aided geometric modeling are normally based on polynomials, since polynomials are simple functions that are easy to calculate and are flexible enough to create many different shapes. However in principle, any functions can be used to create a parametric curve. But, it is really a difficult task to tell much about the behavior of a parametric curve $P(t) = (x(t), y(t))$ by examining the two components $x(t)$ and $y(t)$ separately. Each of the two functions may have features that do not exist in the combination. The reverse is also true that the combined curve may have features not found in any of the two components. The program is written in MATLAB7.10.0. The test result has been shown below in term of Bezier curve.

From the above analysis, the conclusion for this section may be stated as that the parameter "t" is the control room which is the centre part of the spider net(Fig:-6). When the weight function combined with the control points at that moment the potential of this parameter get charged and ready to control the X- and Y- position values. The influence of attraction force on "Y" is more as compared to "X". The controlling power of

weighted function increases with the increase in parameter value "t" from zero to one in an counterclockwise direction. As "t" approaches to 1 (i.e. "t" is going to complete its cycle movement for one time only in a circular path), the power of attraction of control point decreases and becomes to zero at "t=1". This achieves at the last control point of the given control polygon, through which curve passes and ends its motion. During this motion, tangent is the main driver to which are the positions of the control points and are initially taken as the true value filtered from the IGES file. A method to modify a curve for particular degree and order has been discussed through freeform technique. The paper is also focused on the individual behavior of parametric curve components and the net effect of these components on the resulted curve. This research also declares by comparison methods (the coefficient of correlation and rank order comparison) that Bezier curve is 96% of goodness of data fit and have a higher rank as compared to single predicted polynomial trajectory declaration. The total analysis has been carried out through MATLAB-7.10.0 program. At last the potential of the parameter along with the weighted value presents the effect of control point on the curve shape modification through spider technique.

change the direction of the curve in associated with the weight function.

7.CONCLUSION

This paper presents the method to generate a polynomial curve and Bezier spline that passes through the given points in sequence. In this case, the number of unknowns,

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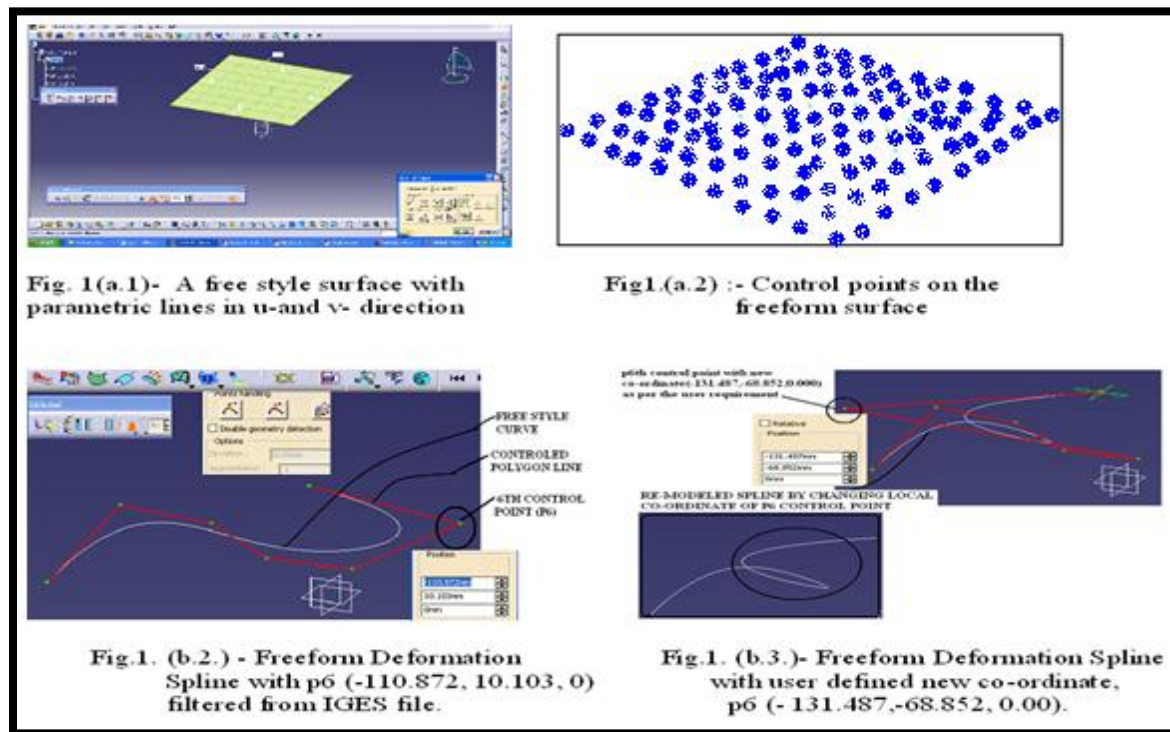
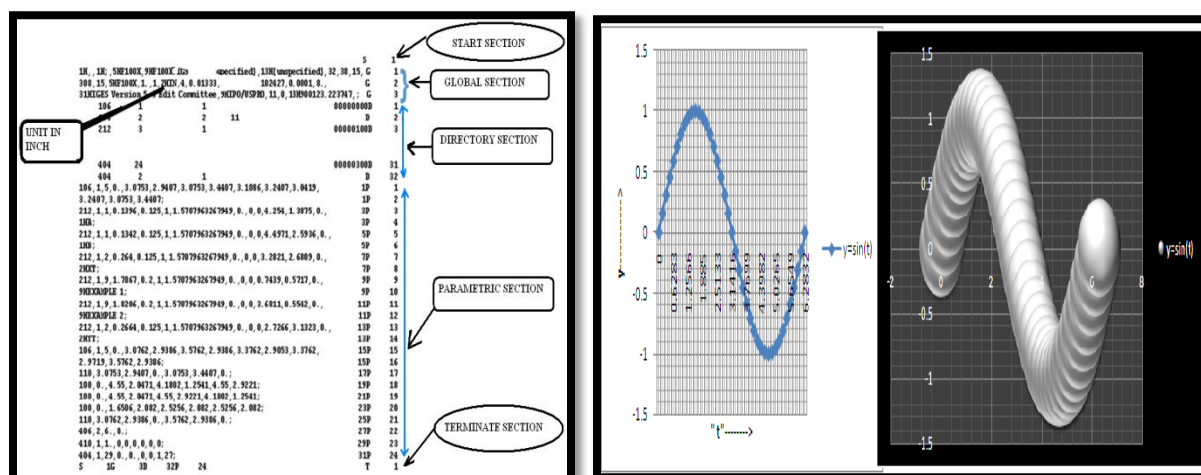


Fig.-1 : Freeform Deformation of spline in relation to surface modeling



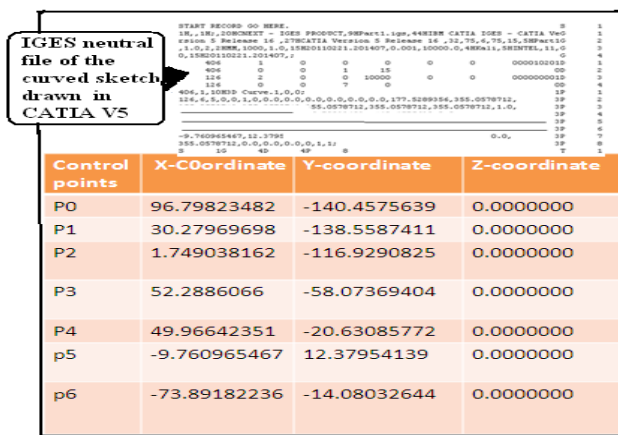
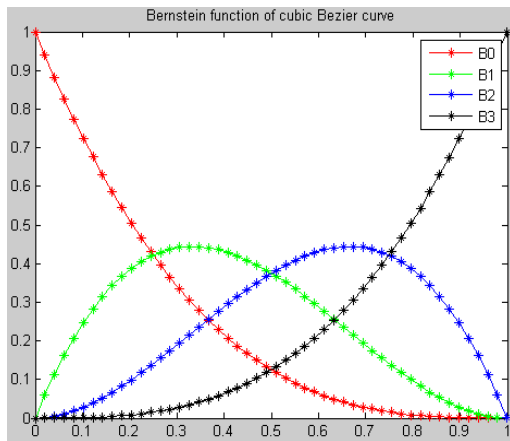


Fig:-3.2 Behavior of Bernstein function Fig:-4 Extracted control points from the .igs file

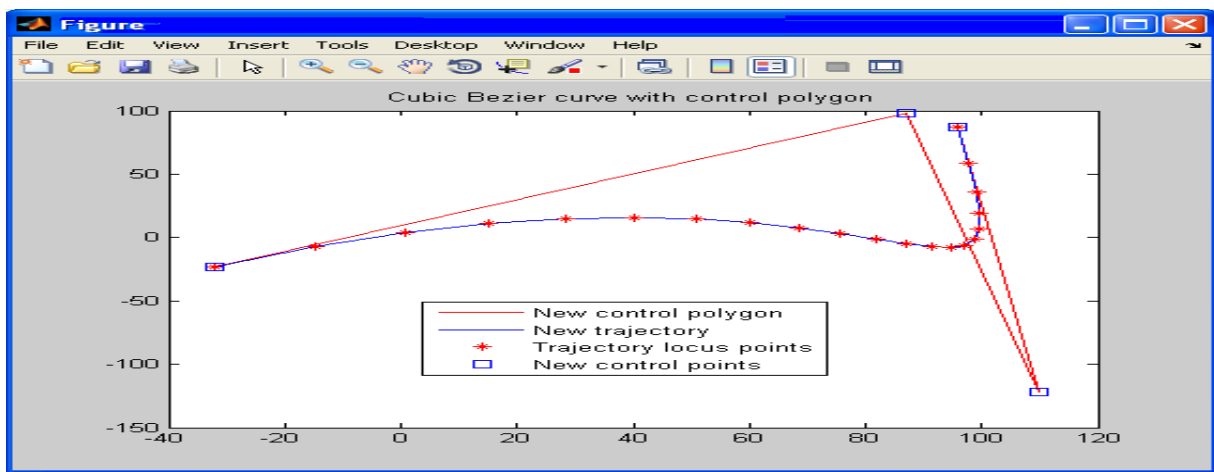


Fig: 5.2. Modified trajectory after free change in main control point $X=[96 \ 110 \ 87 \ -32]$;
 $Y=[87 \ -122 \ 98 \ -23]$; $Z=[0 \ 0 \ 0 \ 0]$;

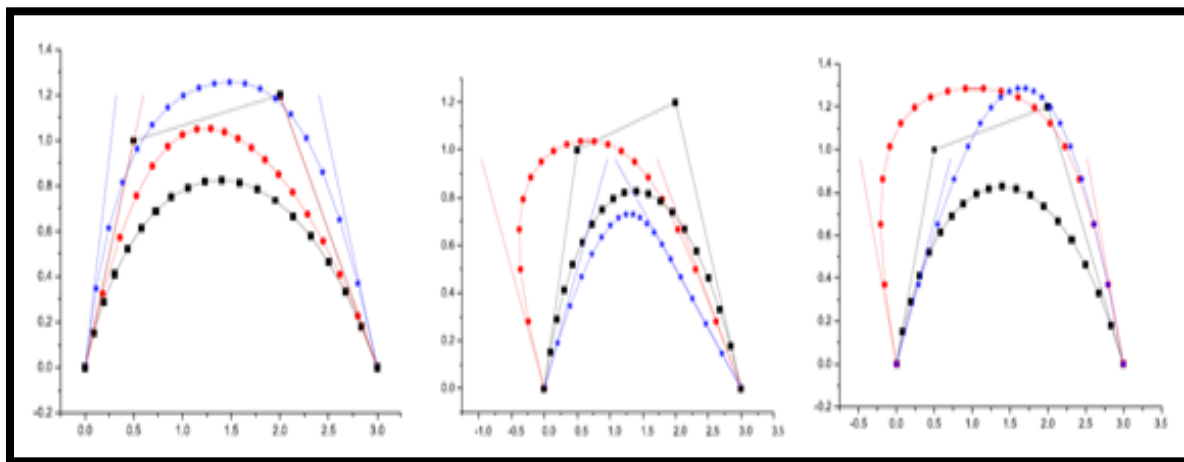


Fig: 5.3. Freeform Bezier curve with modified geometry

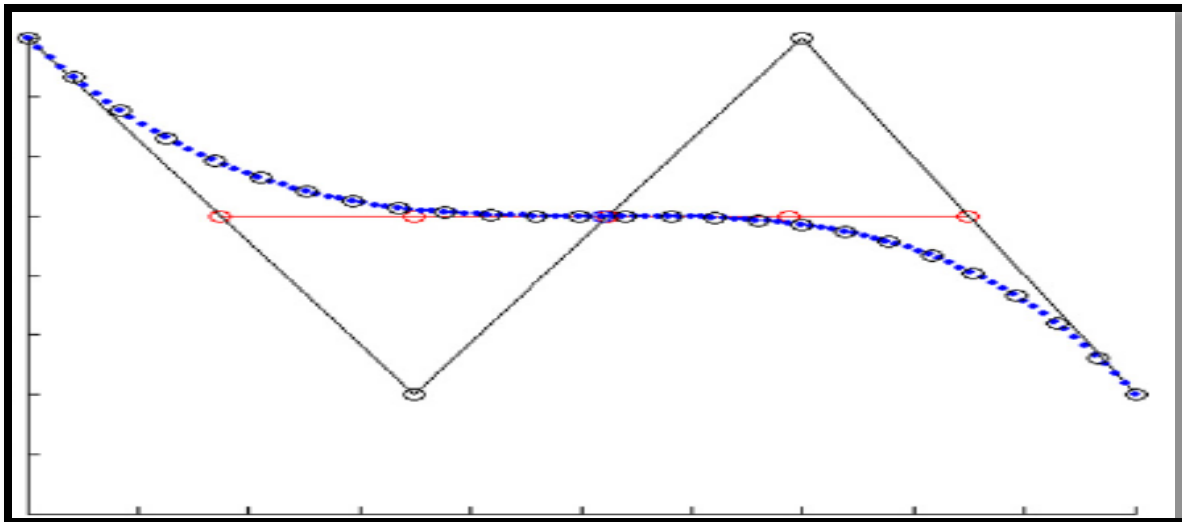


Fig- 5.4 : 26 numbers of generated points for tool trajectory

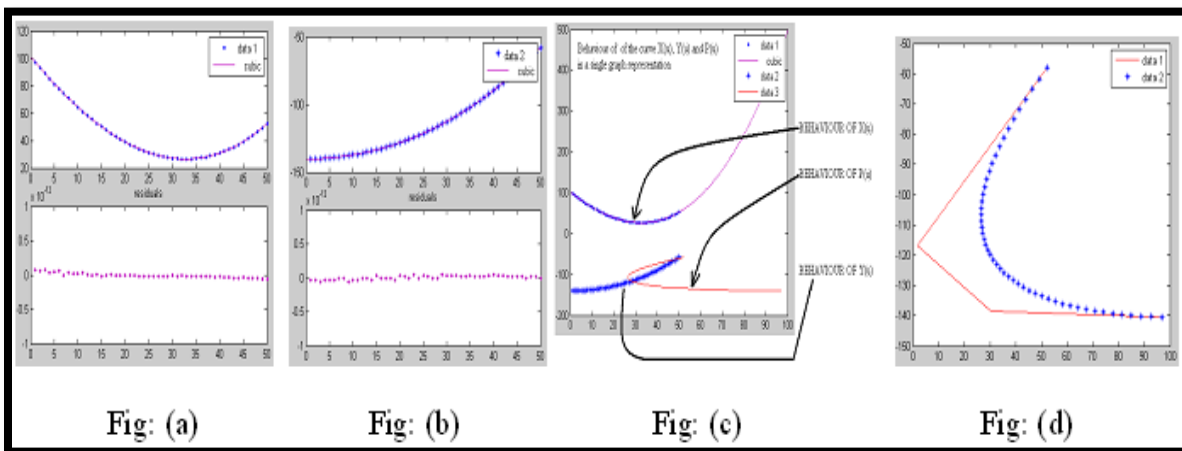


Fig.-6.1: (a) Behavior of $X(u)$ (b) Behavior of $Y(u)$ (c) Behavior of $P(u)$
(d) Cubic Bezier curve $P(u) = (x(u), y(u))$.

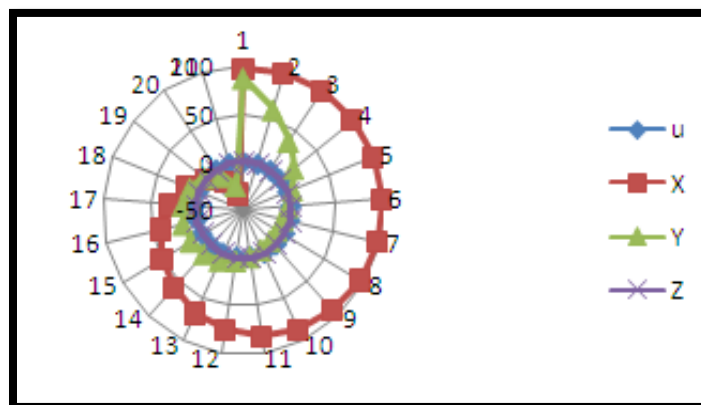


Fig.- 6.2 : Influence of X and Y in reference to the parameter “t” through spider net.

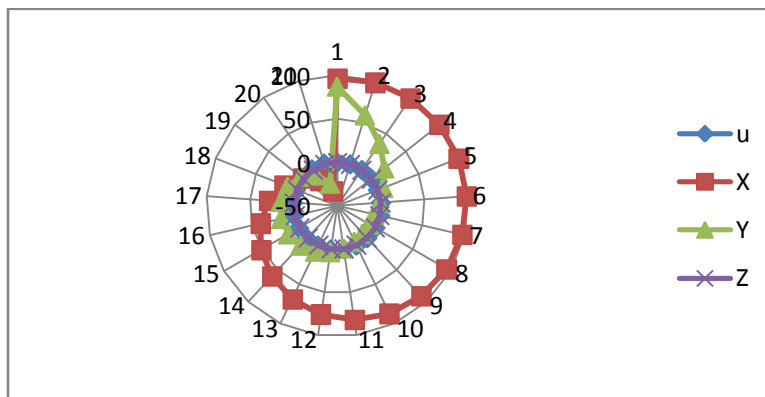


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